PHYSICAL REVIEW E **69**, 045201(R) (2004)

Chaotic behaviors of operational amplifiers

Geo-Su Yim, Jung-Wan Ryu, and Young-Jai Park
Department of Physics, Sogang University, Seoul 121-742, Korea

Sunghwan Rim, Soo-Young Lee, Won-Ho Kye, and Chil-Min Kim
National Creative Research Initiative Center for Controlling Optical Chaos, Pai Chai University, Daejeon 302-735, Korea

(Received 8 October 2003; revised manuscript received 13 February 2004; published 14 April 2004)

We investigate nonlinear dynamical behaviors of operational amplifiers. When the output terminal of an operational amplifier is connected to the inverting input terminal, the circuit exhibits period-doubling bifurcation, chaos, and periodic windows, depending on the voltages of the positive and the negative power supplies. We study these nonlinear dynamical characteristics of this electronic circuit experimentally.

DOI: 10.1103/PhysRevE.69.045201 PACS number(s): 05.45.Ac, 84.30.−r

Chaotic dynamical behaviors have been studied extensively during the last 30 years. Recently these phenomena have attracted much attention for applications to various areas, such as biology, ecology, chemistry, physics, optics, etc. Accordingly, finding an efficient and simple chaos generator is still an essential part of accomplishing various applications. Up to now, many optical [1–6] and electronic [7–10] chaos generators have been introduced.

One class of the simple chaos generators is electronic circuits. The electronic chaos generators, which have been developed up to now, are an inductor-resistor diode [11–15], Chua [7], and inductor-capacitor circuits [16,17]. In the case of the inductor-resistor-diode circuit, the nonlinear capacitance of the diode induces chaos in current and in voltage across the diode. In the case of the Chua circuit, the chaotic behavior of the circuit is induced by nonlinear (piecewise linear) negative resistance designed by an operational amplifier, and is characterized by a double scroll type attractor. And in the case of the inductor-capacitor circuit, the circuit designed by using operational amplifiers consist of four parts, which are the inductor, the negative resistor, the folding circuits, and the linear capacitor. The combination of these four parts generates chaotic behavior of the total circuit. Because of their simplicity, these circuits have been widely used not only for experimental verification of chaos theories but also for the demonstration of recently developed applications such as chaos synchronization, control, and secure communication. In this paper we investigate a new electronic circuit made with an operational amplifier, abbreviated *OP Amp*, which can be a promising candidate for a simple chaos generator.

An OP Amp is one of the important elements in electronics to perform a wide variety of linear operations along with nonlinear operations. It usually has five terminals: a positive (V_+), an inverting (V_-) input, an output (V_o), and positive (V_c) and negative (V_e) power supplies. In a circuit using this OP Amp, when the inverting terminal (V_-) is connected with the output through a resistor, and the positive input terminal (V_+) is connected to the ground (voltage follower circuit), the phase of the output signal is shifted to the input signal due to the resistance-capacitance lag network within the OP Amp [18]. When the phase shift of the total lag network equals or exceeds π, the system can generate a periodic signal.

In this circuit, various nonlinear dynamical behaviors also appear at the output terminal of LF357 (National Semiconductor Co.), depending on the applied voltages of V_c and V_e, even though no input signal is applied to the inverting input V_-. However, no report has been made in spite of their rich nonlinearity, as far as we know.

The circuit used for our experiment is shown in Fig. 1. And the chaotic attractor on voltage and current space is shown in Fig. 2 when the resistor is $R=20\ \Omega$. The figure shows a chaotic folded band attractor. In the detail experiment investigating its rich nonlinear dynamical behaviors, we directly connect the output terminal with the inverting input terminal, and the positive input terminal with the ground. To reduce external noise, we connect 100 nF bypass capacitors between each applied voltage and the ground. Nonlinear dynamical behaviors are investigated as we vary the values of V_c and V_e. We measure the time series of the signal by using a digital storage oscilloscope (LeCroy 9354CL), whose memory size is 2 Mbyte and sampling time is 0.5 nsec for a single channel. The memorized signals are transferred into a personal computer to be analyzed.

Typical examples of the temporal behavior and their spectrums are displayed in Fig. 3, depending on the value of V_c when $V_e=-3.72$ V. Figures 3(a) through 3(g) show a period-1 orbit, a period-2 orbit, chaos, type-I intermittency, period-3 window, another chaos, and type-III intermittency at $V_c=2.0$, 4.3, 5.5, 6.05, 6.2, 8.0, and 9.7 V, respectively. As shown in the spectra on the right-hand side,

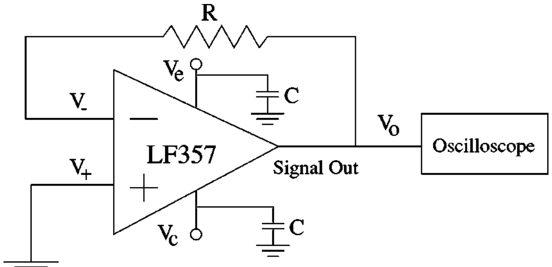

FIG. 1. Schematic diagram of the electronic circuit with an OP Amp that generates chaotic behaviors, where *R* is the effective resistance and *C* is the by-pass capacitor.

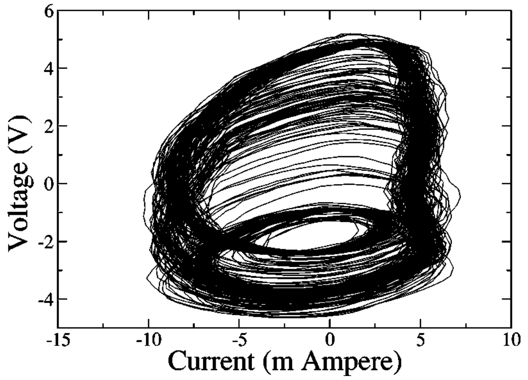

FIG. 2. Chaotic attractor of LF357 OP Amp on the voltage and current space. The current is measured when the 20 Ω resistor is connected.

periodic signals show their subharmonic frequencies and chaotic signal exhibits broad band.

We obtain the bifurcation diagram and the maximal Lyapunov exponent, depending on V_c when $V_e = -3.72$ V.

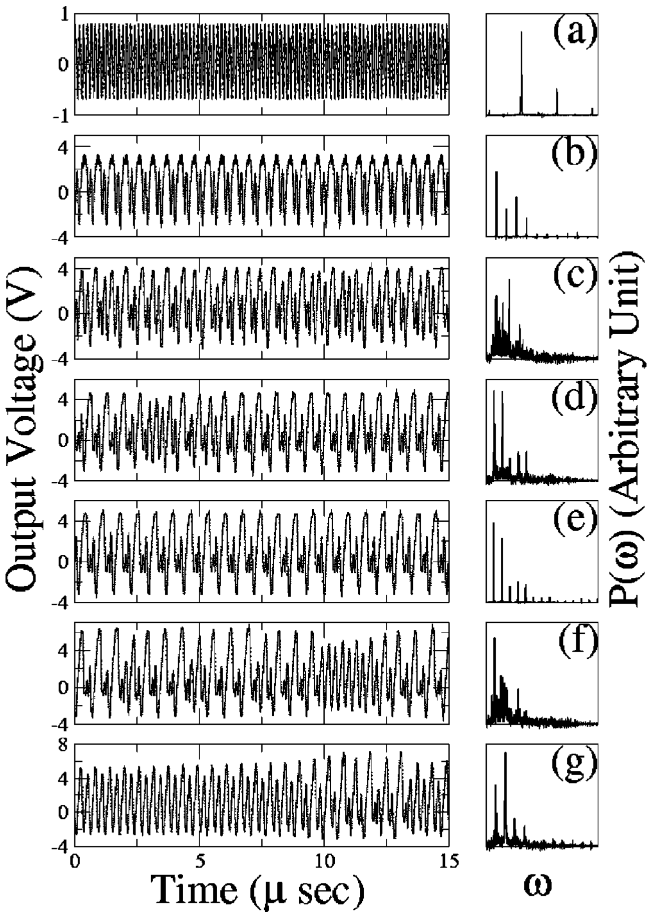

FIG. 3. Various nonlinear dynamical behaviors generated from the simple OP Amp circuit, depending on V_c when $V_e = -3.72$ V: (a) period-1, (b) period-2, (c) chaos, (d) type-I intermittency, (e) period-3, (f) another chaos, and (g) type-III intermittency, when $V_c = 2.0$ V, $V_c = 4.3$ V, $V_c = 5.5$ V, $V_c = 6.05$ V, $V_c = 6.2$ V, $V_c = 8.0$ V, and $V_c = 9.7$ V, respectively. The figures on the left-hand side are temporal behaviors and those on the right-hand side are their spectra.

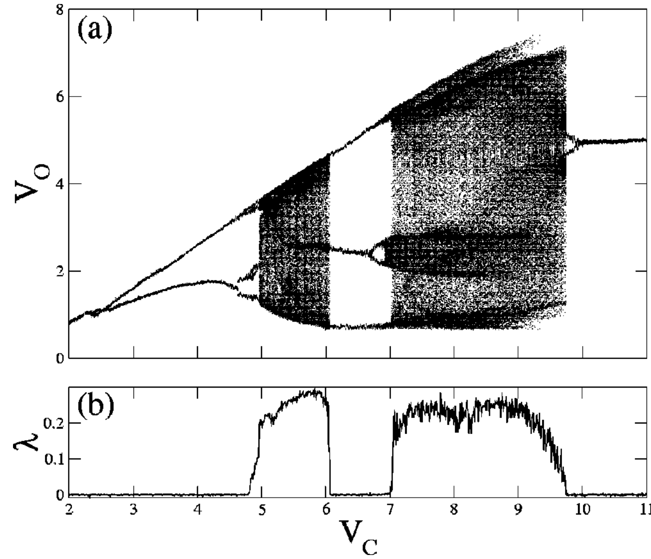

FIG. 4. Bifurcation diagrams (a) and maximal Lyapunov exponent (b) of the LF357 OP Amp, depending on V_c in the range from 2.0 V to 11.0 V at $V_e = -3.72$ V. This figure shows period-doubling bifurcation and periodic windows.

When we obtain the bifurcation diagram, the peak voltage is measured at each period of the output signal V_o. Figure 4(a) exhibits period doubling bifurcation to chaos and a period-3 window, where period-1 transits to period-2 at 2.3 V, to period-4 at 4.6 V, to period-8 at 4.8 V, and so on. The chaotic band transits to a period-3 window through a type-I intermittency at 6.05 V, and the periodic window transits to another wide chaotic band at 7.0 V through period doubling bifurcation. This latter chaotic band transits to period-2 through type-III intermittency at 9.7 V, which transits to period-1 at 9.9 V. Figure 4(b) shows the maximal Lyapunov exponent according to the bifurcation diagram. The value of the exponent is zero when the circuit exhibits a periodic signal. In a chaotic band, the exponent has positive values, which imply the signal is chaotic.

We also obtain the phase diagram in the parameter space of V_c and V_e, as shown in Fig. 5. Here at the normal operation condition of an OP Amp near $V_c = 15.0$ V and $V_e = -15.0$ V, the OP Amp generates a fixed dc voltage that is zero. However, when we vary the voltages of V_c and V_e, we can observe various nonlinear dynamical behaviors such as periods 1,2,4, . . . , periodic windows, and chaos. Near the border of the chaos region, the higher order periodicities are found as given in the bifurcation diagram of Fig. 4, although they are not presented in this phase diagram because of their narrow band. Inside the chaotic region, we also find periodic windows. A similar diagram also appears in the inductor-resistor-diode circuit [19,20]. The straight line at $V_e = -3.72$ is the parametric value where we have obtained the bifurcation diagram shown in Fig. 4. From these data, temporal behaviors, the bifurcation diagram, and the phase diagram, we can see that this simple circuit generates nonlinear dynamical behaviors in a richer variety than the logistic map which is the prototype of the chaos model.

RAPID COMMUNICATIONS

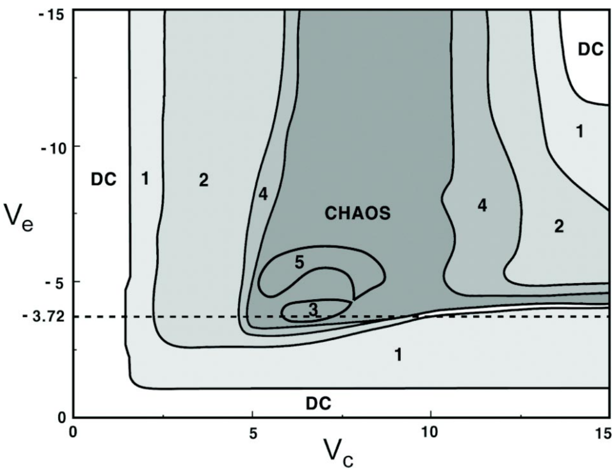

FIG. 5. Phase diagram of the circuit with LF357 in the parameter space of V_c and V_e (volts). Various regions of nonlinear dynamical behaviors such as period-1, period-2, period-4, periodic windows, and chaos are shown.

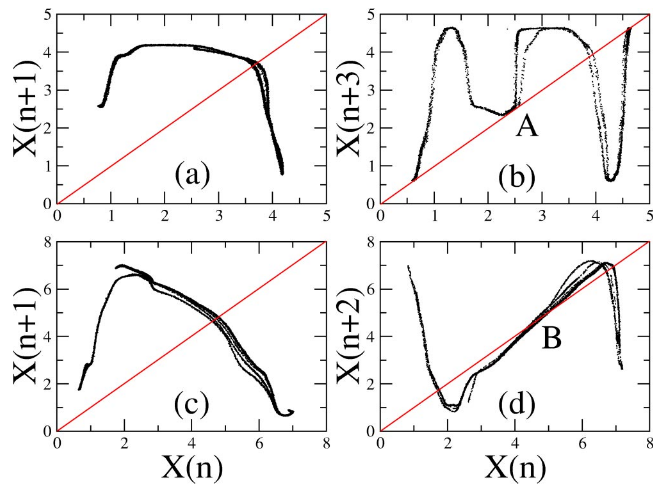

FIG. 6. Return maps of the chaotic behaviors at $V_c = 6.05$ V and $V_c = 9.7$ V: (a) x_n versus x_{n+1} and (b) x_n versus x_{n+3} space for $V_c = 6.05$ V, and (c) x_n versus x_{n+1} and (d) x_n versus x_{n+2} space for $V_c = 9.7$ V. Here V_e is fixed at -3.72 V.

Now, let us obtain return maps at two points in the bifurcation diagram. At $V_c = 6.05$ V, before a period-3 window, the circuit exhibits a simple return map on x_n versus x_{n+1} space as shown in Fig. 6(a) which is similar to that of an inductor-resistor-diode circuit [11–15]. When we obtain the return map on x_n versus x_{n+3} space, the map has a channel between the diagonal line and the local quadratic minimum at point A as shown in Fig. 6(b). Because of this channel, the circuit generates type-I intermittency as shown in Fig. 3(d) [23–25]. At $V_c = 9.7$ V, before the end of the chaotic band we obtain the return map on x_n versus x_{n+1} space as shown in Fig. 6(c). On x_n versus x_{n+2} space the map has a local cubic structure when it crosses the diagonal line as shown in Fig. 6(d). Because of this structure, the circuit generates type-III intermittency [26,27] at $V_c = 9.7$ V. In our experiment, the return maps of the first and the second chaotic bands are similar to those in Figs. 6(a) and 6(c), respectively. This means that the structure of the chaotic attractor is changed as passing the periodic window. In these figures, we also observe a faint splitting of the return maps. This splitting implies that the attractor of the circuit is not a simple but a high-dimensional one, similar to the attractor of an inductor-resistor-diode circuit [21,22]. In our analysis, the embedding dimension of the chaotic attractor is higher than four.

On the other hand, we have investigated the chaotic behaviors of three kinds of OP Amps such as OPA27 (Burr Brown Co.), LF353, and LF412 (National Semiconductor Co.) for generic characteristics of the OP Amp. All of these OP Amps also exhibit different patterns of nonlinear behaviors, including chaos for a certain region of V_c and V_e. And we have observed that each kind of the OP Amps has its own nonlinear dynamical characteristics, such as the bifurcation diagram, the phase diagram in the parameter space of V_c and V_e, and the return map. It has been also observed even OP Amp circuits with the same model of an OP Amp have slightly different chaos and periodic window regions in the parameter space of V_c and V_e due to the slight parameter mismatch among OP Amps in the manufacturing process.

To emphasize, our circuit is basically different from the other chaotic electronic circuits of Chua and inductor-capacitor circuits. While the latters are designed to have a negative resistor using an OP Amp, our circuit generates chaos due to its own characteristic of an OP Amp. So while the other circuits consist of many electronic elements, our circuit has only an OP Amp without any electronic element. In the Chua circuit, the main role generating chaos is the voltage-current curve of the piecewise linear negative resistor. However, in our circuit the total phase shift of the lag network within the OP Amp generates chaos.

In conclusion, we have found rich nonlinear dynamical behaviors in an OP Amp circuit such as period-doubling bifurcation to chaos, periodic windows, and intermittencies. We think that this suprisingly simple circuit can be used for real applications very efficiently owing to the simple implementation of chaos. We again expect this circuit can be widely used for easy experimental verification of various nonlinear dynamical behaviors studied in theory.

This work is supported by Creative Research Initiatives of the Korean Ministry of Science and Technology.

[1] G.D. VanWiggeren and R. Roy, Science **279**, 1198 (1998).
[2] G.D. VanWiggeren and R. Roy, Phys. Rev. Lett. **81**, 3547 (1998).
[3] P. Colet and R. Roy, Opt. Lett. **19**, 2056 (1994).
[4] J.P. Goedgebuer, L. Larger, and H. Porte, Phys. Rev. Lett. **80**, 2249 (1998).
[5] H.D.I. Abarbanel and M.B. Kennel, Phys. Rev. Lett. **80**, 3153 (1998).

[6] J. Garcia-Ojalvo and R. Roy, Phys. Rev. Lett. **86**, 5204 (2001).
[7] T. Matsumoto, L.O. Chua, and M. Komuro, IEEE Trans. CAS. **32**, 797 (1985).
[8] L. Kocarev, K.S. Halle, K. Eckert, L.O. Chua, and U. Parlitz, Int. J. Bifurcation Chaos Appl. Sci. Eng. **2**, 709 (1992).
[9] A.A. Alexeyev and M.M. Green, Int. J. Bifurcation Chaos Appl. Sci. Eng. **7**, 2861 (1997).
[10] U. Parlitz, L. Kocarev, T. Stojanovski, and H. Preckel, Phys. Rev. E **53**, 4351 (1996).
[11] J. Testa, J. Perez, and C. Jeffries, Phys. Rev. Lett. **48**, 714 (1982).
[12] E.R. Hunt, Phys. Rev. Lett. **49**, 1054 (1982).
[13] R.W. Rollins and E.R. Hunt, Phys. Rev. Lett. **49**, 1295 (1982).
[14] S.D. Brorson, D. Dewey, and P.S. Linsay, Phys. Rev. A **28**, 1201 (1983).
[15] C.M. Kim, C.H. Cho, C.S. Lee, J.H. Yim, J. Kim, and Y. Kim, Phys. Rev. A **38**, 1645 (1988).
[16] N.J. Corron and S.D. Pethel, Chaos **12**, 1 (2002).
[17] N.J. Corron, S.D. Pethel, and K. Myneni, Phys. Rev. E **66**, 036204 (2002).
[18] T. L. Floyd, *Basic Operational Amplifiers and Linear Integrated Circuits* (Macmillan, New York, 1994).
[19] F. C. Moon, *Chaotic and Fractal Dynamics, An Introduction for Applied Scientists and Engineers* (Wiley, New York, 1992).
[20] T. Klinker, W. Meyer-Ilse, and W. Lauterborn, Phys. Lett. **101A**, 371 (1984).
[21] C. Jeffries and J. Perez, Phys. Rev. A **26**, 2117 (1982).
[22] H. Ikezi, J.S. deGrassie, and T.H. Jensen, Phys. Rev. A **28**, 1207 (1983).
[23] Y. Pomeau and P. Manneville, Commun. Math. Phys. **74**, 189 (1980).
[24] C.M. Kim, O.J. Kwon, E.K. Lee, and H. Lee, Phys. Rev. Lett. **73**, 525 (1995).
[25] J.H. Cho, M.S. Ko, Y.J. Park, and C.M. Kim, Phys. Rev. E **65**, 036222 (2002).
[26] C.M. Kim, G.S. Yim, J.W. Ryu, and Y.J. Park, Phys. Rev. Lett. **80**, 5317 (1998).
[27] I. Kim, C.M. Kim, W.H. Kye, and Y.J. Park, Phys. Rev. E **62**, 8826 (2000).